Forest Science, Vol. 34, No. 3, pp. 621–633.

The Effect of Eccentricity on the Estimation of Basal Area and Basal Area Increment of Coniferous Trees

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ABSTRACT. A sample of noncircular tree cross-sections was examined to determine basal area and basal area increment. Basal area estimates were calculated with a circular model from single measurements of diameter and from averages of two diameter measurements. Basal area increment estimates were made with several geometric models from single measurements of radial increment and averages of two measurements of radial increment. Commonly used tree cross-sectional area estimates investigated were biased, usually overestimating basal area. Several generalized geometric models of area increment were investigated, but none tested were uniformly supported by the data with accuracy related to the specific increment measurements selected. However, results indicated that either a single increment measurement taken on the minor axis or the shortest increment from the major axis yielded accurate estimates of basal area increment for several of the models tested. FOR. SCI. 34(3):621–633.

ADDITIONAL KEY WORDS. Mensuration, out-of-round, eccentric, cross-sectional area, basal area increment.

THE MOST COMMON DIAMETER MEASUREMENT taken on forest trees is on the main stem, usually at breast height, 4.5 ft above the ground. The diameter at breast height (dbh) is often used for determining tree volume and stand stocking levels. The use of the word "diameter" implies that trees are circular in cross-section (Husch, Miller, and Beers 1982). However, cross-sections are not always round. For example, Williamson (1975), in a study of 806 Douglas-fir stems (*Pseudotsuga mensiesii* [Mirb.] Franco var. menziesii) in California, Washington and Oregon, reported an average eccentricity of 0.88 for stump diameters, where eccentricity is defined as the diameter perpendicular to the longest diameter divided by the longest diameter. Monserud (1979) in a study of 777 felled Douglas-fir stems in Idaho and northwestern Montana reported the average eccentricity at breast height was 0.946 with a standard deviation of 0.04. Kellogg and Barber (1981) found the average eccentricity at breast height was 0.92 for 87 coastal hemlock trees (*Tsuga heterophylla* [Raf.] Sarg.) studied in British Columbia. As part of this study, the average eccentricity at breast height was determined for 100 trees randomly selected from 1039 trees felled for a stem analysis project for mixed conifers in northern California and was found to be 0.937 with a standard deviation of 0.04.

The authors are Associate Professor and Professor, respectively, Department of Forestry and Resource Management, University of California, Berkeley, CA 94720. This research was supported in part by the Northern California Forest Yield Cooperative and McIntire-Stennis Project 3679-MS. Special thanks to James Michael Lukes III and Robin L. Filion for their help incollecting and processing the data. Also, special thanks to Samuel G. Scalise for programming and technical assistance on the Talos™ digitizer at the College of Natural Resources, University of California Berkeley. Manuscript received October 19, 1987.

Reprinted from the *Forest Science*, Vol. 34, No. 3, September 1988

Since the noncircularity of stems can cause bias in estimation of basal area and of basal area increment, this was studied as part of a larger study on tree taper volume and tree growth (Biging 1984; Wensel, Meerschaert, and Biging 1987). Although not a motivation for this study, techniques that produce accurate and precise diameter measurements are important in other types of studies. For example, Gregoire and others (1987) have identified the need for accurate measurements of diameters as requisite for assessment of air pollution impacts on forests.

To investigate the effect of tree shape on accuracy of cross-sectional area, Matern (1956) investigated six noncircular geometric figures including an ellipse and a figure composed of two semi-ellipses. In Matern's study, cross-sectional area was computed using a variety of diameter measurements. The diameter measurements fell into three categories: girth,[1] random diameters,[2] and maximum diameter and minimum diameter.[3] For all cases studied, there was an overestimate of true cross-sectional area when area was estimated using girth or random diameters. One random measurement of diameter with a caliper gave the highest overestimate followed by the arithmetic mean of two readings. Results for measures based on maximum and minimum diameters varied by geometric form of the objects studied. If the cross-section is ellipse-like, these diameter measures yield cross-sectional areas similar to those obtained by girth and random diameters. As the figures become more like a square with rounded corners, the use of maximum diameters[4] yielded large overestimates of true cross-sectional area. In contrast, the use of minimum diameters[5] yielded underestimates. However, in all cases studied, the geometric average gave closer results than did arithmetic averages. In a similar study, Chacko (1961) reported on eccentricity of 111 stems in India. Like Matern, he reported that one random measurement of diameter yielded high overestimates of cross-sectional area, but found that the arithmetic average of the maximum diameter and the diameter perpendicular to it gave even more biased results.

Most of the work on basal area increment determination for noncircular stems has been concerned with minimizing the variance of radial increment estimates from increment cores. Matern (1962) and Siostrzoneki (1958), tested the use from one to four cores for radial growth estimation and reported that two cores, taken from opposite sides of the tree, provided the most efficient estimate in terms of reducing the variance of radial increment estimates. While current studies address the variance of the estimates of increment, they do not directly address the accuracy of basal area increment estimates.

The main objective of this paper is to determine the most accurate tree measurements to obtain unbiased estimates of tree basal area and basal area

[1] Girth diameter is defined as circumference/π.

[2] Random diameters were calculated from the following: one diameter in a random direction; the arithmetic or geometric mean of two diameters—taken in random directions or one diameter in a random direction and the second perpendicular to the first.

[3] Maximum and minimum diameter were defined as follows: the arithmetic or geometric mean of two diameters—the minimum and maximum, the minimum and the diameter perpendicular to it or the maximum and the diameter perpendicular to it.

[4] In this case, an average diameter was computed as the arithmetic or geometric mean of the maximum and minimum diameter, or the maximum diameter and the diameter perpendicular to it.

[5] For this case an average was computed as the arithmetic or geometric mean of the minimum and maximum diameter, or the minimum diameter and the diameter perpendicular to it.

increment. Since diameter increment can vary around the stem, a determination of the number of cores to be taken, their location, and the appropriate geometric formulae for computing increment is required. This is accomplished through an empirical comparison of diameter measurements taken from differing orientations around the stem and their resultant basal area estimates. The accuracy of basal area increment estimates is investigated for a number of geometric models with differing choices of radial growth increments. Techniques that obtain unbiased estimates of cross-sectional area and increment are discussed using an example from data collected from mixed-conifer trees of Northern California.

DATA COLLECTION

Breast height cross-sections were selected from trees felled for a stem analysis project analyzing growth and yield of the mixed conifer forest of California [for a description of the data, see Biging (1984)], but were selected and measured in two distinct ways. The first data set contains sections selected for study because they were noticeably out-of-round. The second data set represents a random selection of sections from the stem analysis project.

For the first portion of the data, 45 sections with an eccentricity ratio of 0.98 or less whose average eccentricity was 0.91 (Table 1a) were selected specifically because they were visibly out-of-round. Inside bark circumference (see Figure 1), maximum, and minimum diameters of each section (see Figure 2) were measured to the nearest 1/16 in. (0.16 cm). Cross-sectional area (inside bark) was measured with a compensating polar planimeter (instrument accuracy within ±0.2% of actual). Ten-year basal area increment was measured by using a planimeter on the inner region of the disk (10 years prior) and subtracting it from the current area (square inches). Circumference[6] for the current period was obtained by measuring the perimeter in a

TABLE 1a. Description of out-of-round data.

Variable	Mean	Std. Dev.	Min.	Max.	N
Cross-sectional area (in.^{2})	92.83	91.47	8.30	399.90	45
Eccentricity[a]	0.906	0.05	0.77	0.98	45
Cross-sectional area increment (in.^{2})	21.99	17.91	5.50	79.30	45
Radial increments (in.)					
i_1	0.68	0.32	0.20	1.73	45
i_2	0.65	0.27	0.23	1.52	45
i_3	0.69	0.25	0.31	1.47	45
i_4	0.83	0.30	0.20	1.72	45
i_5	0.97	0.31	0.42	1.94	45
i_6	0.79	0.27	0.34	1.68	45
i_7	0.66	0.26	0.30	1.51	45
i_8	0.65	0.30	0.29	1.78	45

[a] Eccentricity is defined as the diameter perpendicular to the longest diameter divided by the longest diameter.

[6] Inside bark circumference of tree cross-sections is the perimeter of the convex closure encompassing the tree cross-section (Figure 1).

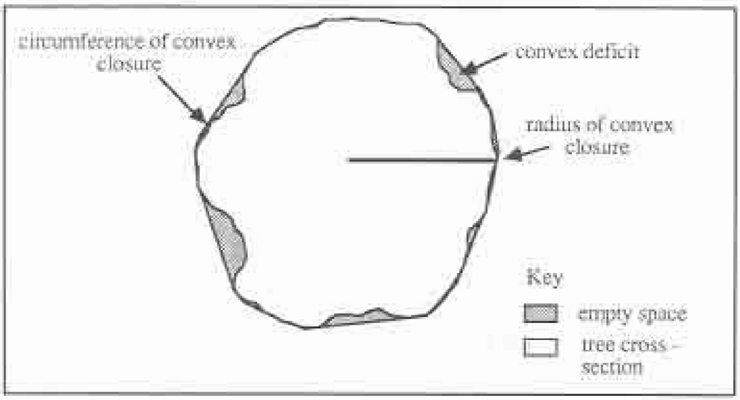

FIGURE 1. Top view of a tree cross-section and its associated circumference.

way which simulates use of a diameter tape (see Figure 1). Additionally, eight 10-year radial increments (denoted $i_1, i_2, \ldots, i_8$) were measured every 45° starting on the major axis. Increment 1 (i_1) is the shortest increment of the major axis, whereas increment 5 (i_5) is the longest increment of the major axis (see Figure 2). The increments were measured to the nearest 1/50 in. (0.5 mm) along an axis through the geometric center of the tree.[7] A summary of these measurements is given in Table 1a. These data are hereafter referred to as the out-of-round data.

The second portion of data contained 50 tree cross-sections and was chosen from photographs taken of breast height sections as part of the same study (cf. Biging and Wensel 1984). These photographs were chosen for

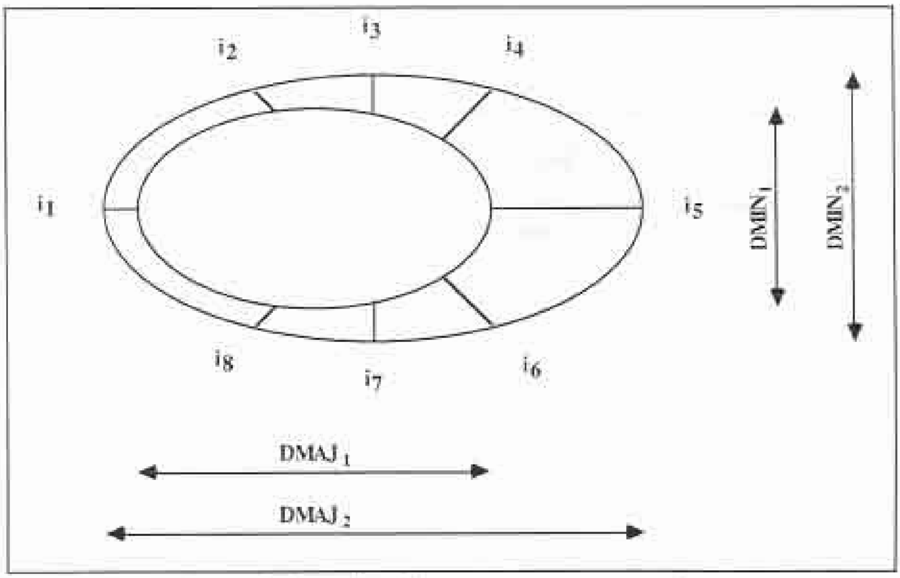

FIGURE 2. Location of increment cores on test sections.

[7] The geometric center is defined as the center of mass or the location of the centroid of the disk under study. For a formal definition refer to calculus texts (cf. eg. Thomas 1966).

clarity of picture and completeness of the section and bark, without regard to eccentricity. These photographs were digitized to determine the same values as for the first portion of the data. An additional measurement of circumference for the prior 10-year period was obtained by digitizing the perimeter in a way which simulates the use of a diameter tape. The accuracy of the digitizing system is approximately 1/1000 in. These measurements are referred to as random data and are summarized in Table 1b.

DATA ANALYSIS

This data set comprised, then, two distinct parts. The first part contained trees that were obviously nonround and the second was comprised of randomly selected trees expected to represent the eccentricity found in the forests sampled. The first data set can be examined to find the effect of being out-of-round on estimates of basal area and growth. The second set is used to interpret the impact that out-of-roundness will likely have on a "typical" selection of sample trees. In the analysis section results will first be presented for the out-of-round data followed by the random data results.

BASAL AREA

The accuracy and precision of cross-sectional area estimates of six models were investigated. They were all of the form $A = \pi R^2$, but varied in the way the radius R was estimated. The six alternative estimators are as follows:

Model	Description	Model Number
$A = \pi\left(\frac{C}{2\pi}\right)^2$	Girth	[1]
$A = \pi\left(\frac{DMAJ}{2}\right)^2$	Longest diameter	[2]
$A = \pi\left(\frac{DMIN}{2}\right)^2$	Shortest diameter	[3]
$A = \pi\left(\frac{DMAJ \times DMIN}{4}\right)$	Geometric mean diameter	[4]
$A = \pi\left(\frac{DMAJ + DMIN}{4}\right)^2$	Arithmetic mean diameter	[5]
$A = \pi\left(\frac{DMAJ^2 + DMIN^2}{8}\right)$	Quadratic mean diameter	[6]

where

A = current cross-sectional area (in.^{2}),

C = current circumference (in.),

$DMAJ$ = current diameter inside bark of the major axis (in.),

$DMIN$ = current diameter inside bark of the minor axis (in.).

The results from testing these models are presented in Table 2. All models were significantly biased for both the out-of-round data and the random data

TABLE 1b. Description of random data.

Variable	Mean	Std. Dev.	Min.	Max.	N
Cross-sectional area (in.²)	161.14	105.57	20.38	521.57	50
Eccentricity*	0.946	0.037	0.852	1.0	50
Cross-sectional area increment (in.²)	56.02	33.68	11.14	149.75	50
Radial increments (in.)					
i_1	1.40	0.59	0.50	3.38	50
i_2	1.41	0.66	0.53	3.49	50
i_3	1.46	0.66	0.53	3.49	50
i_4	1.50	0.52	0.60	2.91	50
i_5	1.62	0.54	0.75	3.32	50
i_6	1.43	0.46	0.57	2.65	50
i_7	1.35	0.51	0.45	3.04	50
i_8	1.37	0.56	0.51	2.95	50

* Eccentricity is defined as the diameter perpendicular to the longest diameter divided by the longest diameter.

sets, with the exception of model [3] with the out-of-round data. Use of the longest diameter (model [2]) resulted in the highest average bias reported (22% and 7% for the two data sets, respectively). Models [4], [5], [6] displayed approximately a 9% and 1% overprediction for these same sets. Model [6] overestimated basal area more than model [5], which overestimated more than model [4] regardless of data sets. This is an expected result since the quadratic mean is always greater than or equal to the arithmetic mean which in turn is greater than or equal to the geometric mean.[8] It is

TABLE 2. Estimated bias in stem basal area estimates using alternative computational models.

	Out-of-Round Data			Random Data		
Model	Average bias (%)	Average bias (in.²)[a]	SE (in.²)[b]	Average bias (%)	Average bias (in.²)	SE (in.²)
[1]	9.5	8.82	1.21	2.87	4.63	0.96
[2]	22.0	20.42	3.52	7.37	11.87	1.76
[3]	−3.3	−3.09 (ns)[c]	1.56	−4.87	−7.85	1.57
[4]	8.4	7.81	1.20	0.98	1.58	0.68
[5]	8.8	8.24	1.27	1.11	1.79	0.67
[6]	9.3	8.67	1.35	1.25	2.01	0.67

[a] Average bias is defined as:

$$\bar{B} = \frac{1}{n} \sum b_i \text{ where } b_i = \text{predicted}_i - \text{actual}_i$$

[b] The bias standard error:

$$SE_{\bar{B}} = \sqrt{\frac{1}{n(n-1)} \sum (b_i - \bar{B})^2}$$

[c] ns denotes that the bias was not statistically significant at $\alpha = 0.05$.

[8] Although not displayed, the results from using harmonic mean diameter, $A = \pi[DMAJ^{-1} + DMIN^{-1}]^{-2}$ were similar to model [4] using the geometric mean diameter. The average bias was 8.01% and 0.85% for the out-of-round and random data, respectively.

worth noting that the geometric mean, equation [4], (which gives an exact area for an ellipse) still gave statistically significant overestimates, but to a much smaller extent in the random data. Thus, the tree cross-sections could not be considered perfectly elliptical for either data set. But, in a practical sense, for the random data, failure to meet this assumption had little impact on basal area, for average biases were under 1.5%.

Applying a diameter tape to the sections to estimate girth implies using model [1]; this overestimated cross-sectional area by 9.5% on the out-of-round data and 3% for the random data. Because a nonconvex region (tree cross-section) has smaller area than its convex closure (Valentine 1976) it is not surprising that model [1], which is based on average diameter of the convex closure, overestimates cross-sectional area. Hence, this work suggests that when sampling eccentric trees, a diameter less than average should yield closer approximations to true area as demonstrated by use of the minor axis (model [3]) for the out-of-round data set that was the least biased model for that data set. Unlike the other models tested, however, it underpredicted basal area. Since the random data were in general less out-of-round, taking a diameter less than average resulted in further underestimation. Thus, this approach should probably be considered only when working with a sample, or population, of trees known to exhibit eccentricity. In situations where the sample is not dominated by eccentric trees, the geometric mean of the major and minor axis was superior to a D-tape determination of area, but both techniques provide acceptable results.

Basal Area Increment

Two classes of geometric models (elliptical and circular) are examined for their accuracy in representing basal area increment of nonround trees. These models are tested with the out-of-round data and the random data described above. Defining the basal area increment, BAI, as the difference between the actual basal areas at two points in time, as shown in Figure 2, four elliptical models and two circular models are investigated.

Elliptical Models

In equations [7] through [10] basal area increment (*BAI*) is computed as the difference between two ellipses. As shown in Figure 2, $DMAJ_2$, $DMIN_2$, $DMAJ_1$, and $DMIN_1$ are the current diameter of the major and minor axis and at the beginning and end of the 10-year growth periods. If the cross-section is a true ellipse, the *BAI* is found by,

$$BAI = \left(\frac{\pi}{4}\right)(DMAJ_2 DMIN_2 - DMAJ_1 DMIN_1) \qquad [7]$$

This equation also computes the *BAI* exactly if the tree is circular.

Three additional models for basal area increment patterned after model [7] are investigated, each having a different assumed geometric configuration. These are depicted in Figures 3, 4, and 5. The first model investigated (see Figure 3) assumes that increment does not vary around the stem. The elliptical *BAI* can be computed from a single increment core taken along either the major or minor axis. As shown in model [8],

$$BAI = \left(\frac{\pi}{4}\right)(DMAJ_2 DMIN_2 - (DMAJ_2 - 2i)(DMIN_2 - 2i) \qquad [8]$$

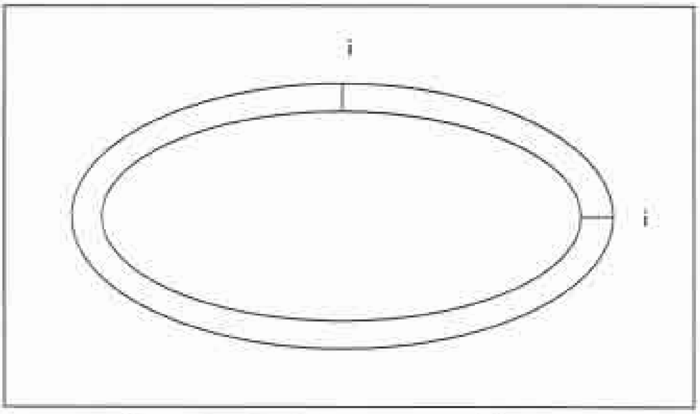

FIGURE 3. Equal increment around the stem.

where

i = the radial increment on the major or minor axis.

The second geometric model investigated assumes that the growth increment is greater along the major axis (i_{MAJ}) than along the minor axis (i_{MIN}) (see Figure 4). In addition, the ellipses are assumed to be symmetric with respect to the geometric center of the tree. Under this assumption the elliptical BAI is given as

$$BAI = \left(\frac{\pi}{4}\right)(DMAJ_2DMIN_2 - (DMAJ_2 - 2i_{MAJ})(DMIN_2 - 2i_{MIN})) \quad [9]$$

The third model investigated assumes that the two ellipses are asymmetric with respect to the geometric center of the tree. The degree to which the inner ellipse is askew is not generally known. However, if we assume that the ratio of major to minor axis stays constant over the growth period, then BAI can be computed using model [10]. In this formulation two increment cores are taken from opposite sides of the tree along the major axis. Denote i_1 and i_5 as 10-year radial increments taken from opposite sides of the major axis (see Figure 5). Then,

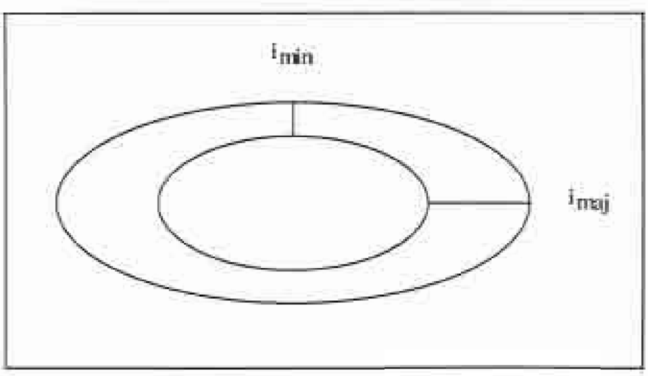

FIGURE 4. Greater increment on major than on minor axis.

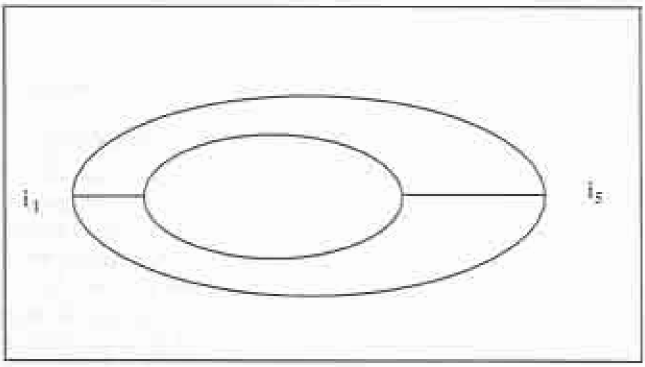

FIGURE 5. Asymmetric radial increment.

$$BAI = \left(\frac{\pi}{4}\right)$$
$$\left(DMAJ_2DMIN_2 - (DMAJ_2 - (i_1 + i_5))\left(DMIN_2 - \left(\frac{DMIN_2}{DMAJ_2}\right)(i_1 + i_5)\right)\right) \qquad [10]$$

In order to assess the bias associated with applying these geometric models to the data (model misspecification error), the estimate of cross-sectional area increment obtained with models [7], [8], [9], and [10] will be compared in accuracy with increment computed as the change in planimetered area. Large bias associated with a model is evidence of the inapplicability of the specified geometric model in representing basal area increment of the sample sections. The use of the various diameter increments, i_1 through i_8, in the elliptical equations [8], [9], and [10] is summarized as follows:

(a) increments i_1, i_3, i_5, and i_7 each were used to compute *BAI* with model [8].
(b) increment pairs (i_1, i_3), (i_1, i_7), (i_5, i_3), (i_5, i_7) were used to compute *BAI* with model [9].
(c) the sum of increments i_1 and i_5 were used to compute increment with model [10].
(d) BAI was computed using caliper measurements of the major and minor axis with model [7].

To examine the validity of the models shown in Figures 3–5, basal area increment was compared to the planimetered area or digitized area under combinations (a), (b), (c), or (d) above. The results of these computations are given in Table 3.

Model [7] can be used for trees calipered at two points in time. When tested against the out-of-round data there was a large average overprediction of 9%, but only 0.5% average overprediction in the random data. Hence, when trees are out-of-round this model can not be supported as an unbiased prediction of basal area increment.

In model [8], it is assumed that the increment does not vary along the stem. If this is the case, any of the increments (i_1, i_3, i_5, or i_7) should give acceptable results. Calculations based on increments of i_1, i_3, or i_7 varied from 2–6% and 1–6% from the actual *BAI* for the two data sets, respectively, but were not statistically significant. However, cross-sectional area computed using the largest increment of the major axis (i_5) was significantly

TABLE 3. Estimated bias in tree basal area increment estimates using alternative radial increment measurements with four elliptical models.

	Out-of-Round Data			Random Data		
Model/ Increments	Average bias (%)	(in.²)[a]	SE (in.²)[b]	Average bias (%)	(in.²)	SE (in.²)
[7]	9.23	2.03	0.35	0.54	0.30 (ns)	1.93
[8] i_1	2.62	0.58 (ns)[c]	0.65	−3.71	−2.08 (ns)	2.29
[8] i_3	1.55	0.34 (ns)	0.89	0.77	0.43 (ns)	2.26
[8] i_5	39.31	8.65	1.19	10.21	5.72	2.08
[8] i_7	−5.50	−1.21 (ns)	0.61	−5.62	−3.15 (ns)	2.16
[9] i_1,i_3	2.14	0.47 (ns)	0.59	−1.21	−0.68 (ns)	2.16
[9] i_1,i_7	−1.50	−0.33 (ns)	0.39	−4.52	−2.53 (ns)	2.07
[9] i_5,i_3	19.92	4.38	0.70	5.53	3.10 (ns)	2.06
[9] i_5,i_7	16.28	3.58	0.46	2.36	1.32 (ns)	1.91
[10] $\bar{i}_1 + i_5$	15.17	3.33	0.51	0.87	0.49 (ns)	2.03

[a] Average bias is defined as:

$$\overline{B} = \frac{1}{n} \sum b_i \text{ where } b_i = \text{predicted}_i - \text{actual}_i$$

[b] The bias standard error:

$$SE_{\overline{B}} = \sqrt{\frac{1}{n(n-1)} \sum (b_i - \overline{B})^2}$$

[c] ns denotes that the bias was not statistically significant at $\alpha = 0.05$.

biased ($\alpha = 0.05$) and averaged 39% in overestimation for the out-of-round data and 10% in the random data. Thus, while model [8] can not be universally accepted as representing basal area increment of nonround stems because of the variation in increment around the stem, it does provide relatively unbiased estimates of basal area increment for all but the most extreme measure of diameter increment.

For model [9], increment is assumed to vary around the stem and the ellipses are symmetric around the geometric center of the tree. It is postulated that increment calculations will be similar whether using increments i_1 and i_3, i_1 and i_7, i_5 and i_3, or i_5 and i_7. However, results show that there is a 18% average overprediction for the out-of-round data when using either of the increment pairs (i_5, i_3) or (i_5, i_7) and only a 4% average bias using the random data. Predictions using the increment pairs (i_1, i_3) or (i_1, i_7) are not significantly biased in either data set. While equation [9] cannot be uniformly accepted as representing increment of nonround trees, well-chosen increments help to provide relatively accurate increment estimates.

In the formulation of model [10], it is assumed that the inner ellipse is not symmetric to the geometric center of the tree. However, the ratio of the increments along the major and minor axes is assumed equal to the ratio of the length of the axes. Results show a 15% difference between actual and predicted increment for the out-of-round data using this model (Table 3), but there was no significant bias when judged against the random data set. Since this difference is statistically significant ($\alpha = 0.05$) on the out-of-round data set, we conclude that the ratio of increment along the major and minor axes is not proportional to the ratio of the major and minor axis for the out-of-round trees, and that this model is inappropriate for representing basal area increment of these trees.

None of the above models could be confirmed by the data, owing to the large average biases associated with cross-sectional area estimated with many combinations of increments investigated. Thus, none of the models postulated is applicable for all combinations of increments used in estimating basal area increment and may result in appreciable model bias (misspecification error).

The use of some of the increments, in conjunction with the current size as measured by calipers, gave basal area increment figures close to those obtained with the planimeter and digitizer values on either data set. The use of increments i_1, i_3, or i_7 in model [8], or the use of increments i_1 and i_3 or i_1 and i_7 with model [9] gave statistically unbiased results regardless of the data set. In general, these increments represent values less than the average around the stem. This helps to compensate for an overestimate of tree basal area when using the geometric mean of the major and minor axis. When one increment core is taken, the use of an increment from the minor axis is superior to choosing an increment from the major axis. Unfortunately, if model [9] is to be employed, three increment borings are needed since increment i_1, the smaller of the two increments on the major axis, cannot be distinguished from i_5 without measuring both, in addition to taking at least one core along the minor axis.

Therefore, when using calipers to measure tree size using only one increment, it is recommended that the increment be taken along the minor axis and used with model [8]. If model [9] is used, three increments need to be taken i_1 and i_5 along the major axis and either i_3 or i_7 along the minor axis. Even though this is a more time-consuming procedure, the average bias associated with this model and choice of increments is less than 10% for out-of-round data and under 3% for the random data.

Circular Models

If the true cross-section is circular, the *BAI* can be expressed as

$$BAI = \pi\left(\frac{C_2}{2\pi}\right)^2 - \pi\left(\frac{C_1}{2\pi}\right)^2 \qquad [11]$$

where C_1 and C_2 denote the circumference at the beginning and end of the growth period, respectively.

Assuming measurements at only one time period, C_2 is directly measurable with a diameter tape. C_1 must usually be estimated from C_2 and some estimate of the radial increment i. Algebraically this can be written as,

$$BAI = \pi\left(\frac{C_2}{2\pi}\right)^2 - \pi\left(\frac{C_2}{2\pi} - i\right)^2 \qquad [12]$$

For model [11], it is assumed that basal area increment is the difference in areas determined with a diameter tape. The results for the random data are listed in Table 4 and were among the least biased of all basal area increment models tested.

Table 4 gives the *BAI* using equation [12] with each of the individual increments i_1 through i_8 as well as the geometric mean of 12 combinations of the original 8 increments. When using only one increment core for model [12], increments i_1, i_2, i_3, i_7 and i_8 display no significant bias. These results hold for both data sets. However, increments i_1, i_2 and i_8 are difficult to distinguish from their symmetric counterparts (i_5, i_6 and i_4, respectively)

TABLE 4. Estimated bias in tree basal area increment estimates using alternative measurements of radial increment with two circular models.

	Out-of-Round Data			Random Data		
Model/ Increment	Average bias (%)	Average bias (in.²)[a]	SE (in.²)[b]	Average bias (%)	Average bias (in.²)	SE (in.²)
[11]	na[c]	na	na	−0.50	−0.28 (ns)	1.30
[12] i_1	3.07	0.67 (ns)[d]	0.64	−2.83	−1.59 (ns)	2.27
[12] i_2	−2.91	−0.64 (ns)	0.59	−1.68	−0.94 (ns)	2.41
[12] i_3	2.00	0.44 (ns)	0.87	1.91	1.07 (ns)	2.25
[12] i_4	21.61	4.75	1.01	3.05	1.71 (ns)	2.20
[12] i_5	40.03	8.80	1.21	11.60	6.50	2.09
[12] i_6	14.67	3.23	0.59	0.23	0.13 (ns)	1.79
[12] i_7	−5.07	−1.12 (ns)	0.60	−4.80	−2.69 (ns)	2.11
[12] i_8	−4.45	−0.98 (ns)	0.65	−4.14	−2.32 (ns)	2.09
[12] $\sqrt{i_1 i_3}$	1.85	0.41 (ns)	0.56	−0.78	−0.44 (ns)	2.13
[12] $\sqrt{i_1 i_5}$	19.45	4.28	0.68	3.84	2.15 (ns)	2.02
[12] $\sqrt{i_1 i_7}$	−1.87	−0.41 (ns)	0.36	−4.19	−2.35 (ns)	−1.43
[12] $\sqrt{i_2 i_4}$	7.81	1.71	0.43	0.32	0.18 (ns)	2.13
[12] $\sqrt{i_2 i_6}$	4.91	1.08	0.31	−1.23	−0.69 (ns)	1.87
[12] $\sqrt{i_2 i_8}$	−4.08	−0.90 (ns)	0.53	−3.16	−1.77 (ns)	2.16
[12] $\sqrt{i_3 i_5}$	18.99	4.18	0.71	6.39	3.58	0.004
[12] $\sqrt{i_3 i_7}$	−2.62	−0.58 (ns)	0.47	−2.10	−1.18 (ns)	1.92
[12] $\sqrt{i_4 i_6}$	17.72	3.90	0.73	1.34	0.75 (ns)	1.87
[12] $\sqrt{i_4 i_8}$	6.82	1.50	0.41	−1.09	−0.61 (ns)	1.95
[12] $\sqrt{i_5 i_7}$	14.75	3.24	0.43	2.68	1.50 (ns)	1.88
[12] $\sqrt{i_6 i_8}$	4.17	0.92	0.41	−2.43	−1.36 (ns)	1.73

[a] Average bias is defined as:

$$\overline{B} = \frac{1}{n} \sum b_i \text{ where } b_i = \text{predicted}_i - \text{actual}_i$$

[b] The bias standard error:

$$SE_{\overline{B}} = \sqrt{\frac{1}{n(n-1)} \sum (b_i - \overline{B})^2}$$

[c] For the out-of-round data the prior circumference was not recorded and, hence this calculation is not available.

[d] ns denotes that the bias was not statistically significant at $\alpha = 0.05$.

which display significant bias on the out-of-round data. Of these three (i_5, i_6 and i_4) only i_5 displays significant bias on the random data. Thus selecting an increment from the minor axis eliminates coring two increments to identify the one of interest.

If the geometric average of two increment cores is used with model [12], no statistically significant bias was observed in either data set using the increment pairs (i_1, i_3), (i_1, i_7) and (i_3, i_7). Similarly, no significant bias was obtained from the increment pair (i_2, i_8) on the out-of-round data or for the increment pairs (i_2, i_6), (i_2, i_8) and (i_6, i_8) on the random data.

Using the increment pairs (i_1, i_3) or (i_1, i_7) would require a third increment (i_5) to distinguish the increments on the major axis. The increment pairs (i_2, i_6), (i_2, i_8) and (i_6, i_8) cannot be distinguished from their symmetric counterparts without examining both pairs. This fact, coupled with the statistically insignificant bias associated with using increments on the minor axis, regardless of data set, implies that when selecting two increment cores they should be on the minor axis. However, there is only a slight improvement over using a single increment from the minor axis.

CONCLUSIONS AND RECOMMENDATIONS

This study found that simple circular or elliptical models consistently overestimate tree basal area. For nonround trees a diameter less than average, such as length of the minor axis, proved a better prediction of basal area. In situations where the sample is not dominated by eccentric trees, models using means (harmonic, arithmetic, or quadratic) of the major and minor axis diameter were superior to a D-tape determination of area, but neither technique yielded any significant bias.

In the second part of this study, both elliptical and circular model types for predicting basal area increment were investigated. The elliptical models were based on using calipers for diameter measurements while the circular models utilized diameter tapes. For the elliptical models examined, the accuracy was dependent upon the number of increment cores chosen and their location around the stem, but was unbiased if increment along minor axis was used. For the circular models investigated, selecting one or two increment cores from the minor axis provided unbiased estimates of basal area increment.

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